International Journal of Theoretical Physics, Vol. 29, No. 9, 1990

Nonstatic Axisymmetric Universe Model and Its Evolution

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Received February 15, 1990

An axisymmetric rotating universe model is crossed with a time-dependent factor. Its evolution is studied. It turns out that for a particular choice it behaves exactly like that of a flat Friedmann universe.

1. INTRODUCTION AND RESULTS

Static cosmological models do not allow expansion of the universe, whereas in evolving universe models, gravitational sources are not detectable. During the past few years it has been proved rigorously that unlike the Einstein–Strauss model (Misner *et al.*, 1973; Lindquist and Wheeler, 1957), it is in fact possible to construct model universes in which both the effects of gravitation and expansion are detectable simultaneously (Bokhari, 1985; Bokhari and Qadir, 1986). To be able to construct these model universes, the Schwarzschild metric was considered and crossed with the Friedmann universe by a scale factor $a(t)$. It turned out that with respect to a particular choice of evolution the expansion of the universe behaves like the flat Friedmann universe and at the same time the gravitational force experienced by a test particle comes out to be exactly the same as the classical gravitational force due to a point source.

In this paper we extend this analysis to an expanding axisymmetric cosmological model with a rotating gravitational field. This analysis is confined only to seeing how this model universe expands with time. It turns out that, corresponding to a particular choice of the evolution, the expansion turns out to be that of the flat Friedmann universe model.

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0020-7748/90/0900-1011$06.00/0

We consider a nonstatic axisymmetric cosmological model whose metric is given by (Bokhari, 1990; Tiwari *et al.*, 1986)

$$ds^2 = (e^\lambda - e^\rho\omega^2)\,dt^2 - a^2(t)(e^\mu\,dr^2 + e^\nu\,d\theta^2 + e^\rho\,d\phi^2) + 2a(t)e^\rho\omega\,dt\,d\phi \quad (1)$$

where λ, ρ, μ, and ν are functions of r and θ coordinates and ω is the constant angular velocity of the gravitational field. Notice that there does not exist any timelike solution of the Killing equations for this particular model universe. However, choosing a particular scale (conformal) factor Ω which transforms the metric $g_{ab} \to \tilde{g}_{ab} = \Omega^2 g_{ab}$ by $\Omega = a^{-1}(t)$ (Bokhari, 1990), the conformally transformed metric admits a timelike conformal Killing vector $\tilde{k}^a = a(t)\delta_0^a$. This conformal factor so chosen does not disturb the spatial symmetry. Thus, the spatial conformal Killing vector $\tilde{k}^a = \delta_3^a$ remains the same as the spatial Killing vector.

To be able to see the evolution of this model universe, we use the Einstein field equations

$$R_{ab} - \tfrac{1}{2}g_{ab}R = \kappa T_{ab} \qquad (a, b = 0, 1, 2, 3) \quad (2)$$

where

$$R_{ab} = 2(\{a^c[b\}, c] + \{f^c[c\}\{a]^f b\}) \quad (3)$$

and

$$R = g^{ab}R_{ab} \quad (4)$$

to generate the stress-energy tensor. These equations are solved to obtain all the components of the stress-energy tensor. For simplicity we use units in which $c = G = 1$. Using equations (1)-(4), it is easily seen that

$$\begin{aligned} T_{00} = {} & 3\frac{\dot{a}^2(t)}{a^2(t)} + \frac{\omega^2}{a^2(t)} T_{33} \\ & - \frac{e^{\lambda-\mu}}{4a^2(t)}[2(\nu+\rho)_{,11} + \rho_{,1}(\rho-\mu+\nu)_{,1} + \nu_{,1}(\nu-\mu)_{,1}] \\ & - \frac{e^{\lambda-\nu}}{4a^2(t)}[2(\mu+\rho)_{,22} + \rho_{,2}(\rho-\nu+\mu)_{,2} + \mu_{,2}(\mu-\nu)_{,2}] \end{aligned} \quad (5)$$

$$T_{01} = \frac{\dot{a}(t)}{a(t)}\lambda_{,1} \quad (6)$$

$$T_{02} = \frac{\dot{a}(t)}{a(t)}\lambda_{,2} \quad (7)$$

$$T_{03} = -\frac{\omega}{a(t)} T_{33} \quad (8)$$

$$T_{11} = \tfrac{1}{4}[\lambda_{,1}(\nu+\rho)_{,1} + \rho_{,1}\nu_{,1}] - e^{\mu-\lambda}[2a(t)\ddot{a}(t) + \dot{a}^2(t)]$$
$$+ \frac{e^{\mu-\nu}}{4}[2(\lambda+\rho)_{,22} + \lambda_{,2}(\rho-\nu+\lambda)_{,2} + \rho_{,2}(\rho-\nu)_{,2}] \qquad (9)$$

$$T_{12} = \tfrac{1}{4}[-2(\rho+\lambda)_{,12} + \mu_{,2}(\rho+\lambda)_{,1} + \rho_{,2}(\nu-\rho)_{,1} + \lambda_{,2}(\nu-\lambda)_{,1}] \qquad (10)$$

$$T_{13} = 0 \qquad (11)$$

$$T_{22} = \tfrac{1}{4}[\lambda_{,2}(\mu+\rho)_{,2} + \rho_{,2}\mu_{,2}] - e^{\nu-\lambda}[2a(t)\ddot{a}(t) + \dot{a}^2(t)]$$
$$+ \frac{e^{\nu-\mu}}{4}[2(\lambda+\rho)_{,11} + \lambda_{,1}(\rho-\mu+\lambda)_{,1} + \rho_{,1}(\rho-\mu)_{,1}] \qquad (12)$$

$$T_{23} = 0 \qquad (13)$$

$$T_{33} = -e^{\rho-\lambda}[2a(t)\ddot{a}(t) + \dot{a}^2(t)]$$
$$+ \frac{e^{\rho-\mu}}{4}[2(\nu+\lambda)_{,11} + \lambda_{,1}(\nu-\mu+\lambda)_{,1} + \nu_{,1}(\nu-\mu)_{,1}]$$
$$+ \frac{e^{\rho-\nu}}{4}[2(\mu+\lambda)_{,22} + \lambda_{,2}(\mu-\nu+\lambda)_{,2} + \mu_{,2}(\mu-\nu)_{,2}] \qquad (14)$$

Corresponding to the above stress-energy tensor, the model universe considered may evolve in any fashion. To be able to have a naive understanding of the evolution, we choose the one in which

$$2a(t)\ddot{a}(t) + \dot{a}^2(t) = 0 \qquad (15)$$

This equation can be easily solved to give

$$a(t) = (\alpha t + \beta)^{2/3} \qquad (16)$$

where α, β are constants of integration. By appropriately rescaling the origin of the coordinates, equation (16) yields

$$a(t) = \gamma t^{2/3} \qquad (17)$$

where γ is a constant of integration. Now notice that corresponding to the evolution in which equation (15) is satisfied, the model universe whose metric is given by equation (1) evolves as $a(t) \sim t^{2/3}$. By comparing this evolution with the flat Friedmann evolution, it turns out that this model universe has at least one evolution which goes exactly as the flat Friedmann evolution.

Note that the stress-energy tensor also carries momentum components which should not be present in a realistic cosmological model. To get rid of these terms, we diagonalize the stress-energy tensor by solving an eigenvalue problem. It is easy to see that this gives rise to a cubic equation in λ times $(\lambda - T_{22})$. These roots of this equation with $\lambda = T_{22}$ will give the new required frame in which there will be no momentum term attached with the stress-energy tensor.

2. SUMMARY AND DISCUSSION

A nonstatic axisymmetric model universe is considered. The stress-energy tensor is generated using the Einstein field equations. This stress-energy tensor is quite an unrealistic stress tensor, as it has all the off-diagonal terms. Thus, the model universe so constructed may evolve in any random manner. To be able to have some idea of its evolution, we restrict ourselves to the one in which a combination of the scale factor $a(t)$ is put equal to zero, e.g., see equation (15). This equation is solved. It turns out that this choice of evolution yields the same evolution as that of the flat Friedmann universe.

The appropriate frame with no momentum terms has been obtained by a diagonalization procedure. If the analysis had been extended to the force expression, it would have yielded new insights into the working of this model. This question, however, remains unanswered. Also, comparing this analysis with the one in Bokhari and Qadir (1986) and Bokhari (1990), it is conjectured that whatever cosmological model one constructs [crossing by a scale factor $a(t)$], it would yield a flat Friedmann universe as one of its evolutions.

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